

**A major purpose of the Technical Information Center is to provide the broadest dissemination possible of information contained in DOE's Research and Development Reports to business, industry, the academic community, and federal, state and local governments.**

**Although portions of this report are not reproducible, it is being made available in microfiche to facilitate the availability of those parts of the document which are legible.**

LA-UR--87-660

DE87 006063

TITLE: MODIFICATION OF ACCELERATION ELEMENT IN "TRANSPORT"

AUTHOR(S): J. W. Hurd, MP-5  
J. McGill, MP-13SUBMITTED TO: 1987 Particle Accelerator Conference, Accelerator Engineering  
and Technology, March 16-19, 1987, Washington, D.C.

## DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

By acceptance of this article, the publisher recognizes that the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or to allow others to do so, for U.S. Government purposes.

The Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy.



Los Alamos National Laboratory  
Los Alamos, New Mexico 87545

# MODIFICATION OF ACCELERATION ELEMENT IN "TRANSPORT"

J.W. Hurd, J. McGill, MS H812

Los Alamos National Laboratory, Los Alamos, NM 87544

## Abstract

One of the uses at LAMPF of the beam-dynamics code TRANSPORT is to model and tune the proton beams in the side-coupled linac. Significant differences are found between measured beam characteristics and those calculated by an early version of TRANSPORT. These differences are reduced to acceptable limits by modification of the original accelerator transformation matrix used in the code. The modified element and a brief description of its derivation are presented. The modified matrix reduces to the original matrix in the relativistic limit. The modified matrix is also compared to results of numerical integration to obtain some indication of its accuracy.

## Introduction

Beam behavior predicted by the earlier version of TRANSPORT<sup>1</sup> does not agree with experimental results. Investigation of this problem shows that the original accelerator matrix used in TRANSPORT is valid only for beams with large velocities ( $\beta$  close to 1). For the beam at LAMPF,  $\beta \leq 0.84$ , which is below the energy where the original accelerator matrix is valid. To remedy this problem, a new first-order transformation matrix for accelerator sections is developed. This matrix is designed to use the same input parameters as the earlier TRANSPORT code. There is a slight difference in the definition of the longitudinal bunch-length parameter on which the new accelerator matrix acts. This difference will be described later.

The new transformation through the accelerator section is compared to results of numerical integration of particle motion to validate the accuracy of the new matrix. Second-order terms are estimated from the numerical results for a specific case.

The new accelerator matrix is installed in LAMPF's version of TRANSPORT. The results of the code now fit the measured data to a degree comparable with measurement errors. Satisfactory results have been obtained for studies of both beam centroids and widths.

## Derivation of new Accelerator Matrix

The phase-space parameters used to describe a particle are relative to the on-axis, on-energy synchronous particle. Values with subscript s refer to the synchronous particle. To derive the equations of motion the standard TRANSPORT coordinates  $x, x', y, y'$  are used in the transverse plane. In the longitudinal plane, the difference in energy,  $\Delta W$ , and the difference in phase,  $\Delta\phi$ , are used in the equations of motion. The subscript o refers to the initial values of a parameter, and subscript f refers to the final values.

The fields in an accelerator may be approximated by continuous fields over the length of the accelerator<sup>2</sup>. For a particle near the synchronous particle, the fields are approximated by

$$E_s = \frac{1}{q} \Delta E \cos\phi, \quad E_r = \frac{\gamma_s}{q} \left( \frac{\pi r}{\gamma_s \beta_s \lambda} \right) \Delta E \sin\phi,$$

$$\text{and } B_s = \frac{\gamma_s \beta_s}{qc} \left( \frac{\pi r}{\gamma_s \beta_s \lambda} \right) \Delta E \sin\phi$$

where  $q$  is the charge,  $\gamma, \beta$ , and  $c$  are the normal relativistic terms,  $\lambda$  is the rf wavelength,  $\Delta E$  corresponds to the energy

gain, and  $\phi$  is the longitudinal phase of the particle. Care must be taken in the definitions of  $\phi$  and  $\Delta E$ . To correspond with the original definitions used in TRANSPORT,  $\Delta E \cos\phi_s$  is the energy gained by the synchronous particle over the accelerator section and  $\phi_s$  is positive for a phase stable accelerator.

The approximate equation of motion in the transverse plane is

$$\frac{d}{dz} \gamma_s \beta_s \frac{d}{dz} x = \left( \frac{\pi \Delta E \sin\phi_s}{\lambda L mc^2} \right) \frac{1}{\gamma_s^2 \beta_s^2} x$$

where  $mc^2$  is the rest energy and  $L$  is the length of the accelerator section.

The approximate solutions for particles sufficiently close to the synchronous particle are

$$x_f = \left( \frac{\gamma_{so} \beta_{so}}{\gamma_{sf} \beta_{sf}} \right)^{\frac{1}{2}} \left[ \left( \cosh(k_T L) + \frac{\delta}{k_T} \sinh(k_T L) \right) x_o + \left( \frac{1}{k_T} \sinh(k_T L) \right) x_o' \right], \quad \text{and}$$

$$x_f' = \left( \frac{\gamma_{so} \beta_{so}}{\gamma_{sf} \beta_{sf}} \right)^{\frac{1}{2}} \left[ k_T \left( 1 - \frac{\delta^2}{k_T^2} \right) \sinh(k_T L) x_o + \left( \cosh(k_T L) - \frac{\delta}{k_T} \sinh(k_T L) \right) x_o' \right]$$

where

$$\delta = \frac{1}{2} \frac{W_{so} \Delta E \cos\phi_s}{L p_{so}^2 c^2}, \quad A^2 = \frac{\pi (mc^2)^2 \Delta E \sin\phi_s}{\lambda L p_{so}^2 c^3},$$

$$k_T = \sqrt{\delta^2 + A^2},$$

$W_{so}$  is the initial energy of the synchronous particle, and  $p_{so}$  is initial momentum.

In the longitudinal plane, the approximate equations of motion are given by

$$\frac{d\Delta W}{dz} = -\frac{\Delta E \sin\phi_s}{L} \Delta\phi \quad \text{and} \quad \frac{d\Delta\phi}{dz} = \frac{2\pi}{\lambda} \frac{\Delta W}{\gamma_s^2 \beta_s^2 mc^2}$$

with a solution, again an approximation for particles sufficiently close to the synchronous particle, given by

$$l_f = \left( \frac{\beta_{so} l}{\beta_{so}} \right) \left[ \cos(k_L L) l_o + \left( \frac{1}{\gamma_{so}^2 k_L} \sin(k_L L) \right) \frac{\Delta p_o}{p_o} \right],$$

and

$$\frac{\Delta p_f}{p_o} = \left( \frac{\gamma_{so} \beta_{so}^2}{\gamma_{sf} \beta_{sf}^2} \right) \left[ -\gamma_{so}^2 k_L \sin(k_L L) l_o + \cos(k_L L) \frac{\Delta p_o}{p_o} \right],$$

where

$$k_L = \left[ \frac{2\pi (mc^2)^2 \Delta E \sin\phi_s}{\lambda L p_{so}^2 c^3} \right]^{\frac{1}{2}}$$

The phase-space coordinates  $\Delta W$  and  $\Delta\phi$  are related to the coordinates used by TRANSPORT by the relations

$$l = \frac{\beta_s \lambda}{2\pi} \Delta\phi, \quad \text{and} \quad \frac{\Delta p}{p_s} = \frac{1}{\beta_s^2} \frac{\Delta W}{W_s} \quad \text{for} \quad \frac{\Delta p}{p_s} \ll 1.$$

There is a small difference in the definition of the bunch-length parameter  $l$  and the corresponding parameter used in TRANSPORT. The parameter  $l$  used in this paper is the difference in longitudinal position of a particle from the synchronous particle at a specific point in time. The standard parameter  $l$  used in TRANSPORT is the difference between the path length followed by a particle and the path length followed by the synchronous particle across a given length.

#### Resulting Transformation

The resulting non-zero matrix elements are shown in Table 1 and compared to original elements used in TRANSPORT. These new elements reduce to the original elements in the limit as  $\beta \rightarrow 1$ . They also reduce to the standard drift matrix elements for  $\Delta E = 0$ .

The transformation can also be compared to results of numerical integration for specific cases. To obtain the numerical results, a six-dimensional random Gaussian distribution is generated. The distribution function is given by

$$f(\bar{x}) = \frac{1}{(2\pi)^{\frac{1}{2}} \epsilon_x \epsilon_y \epsilon_z} e^{-\frac{1}{2} (\bar{x}^T \sigma^{-1} \bar{x})}$$

where  $\bar{x}$  is the phase-space coordinate vector,  $\sigma$  is the standard beam sigma matrix and  $\epsilon_x, \epsilon_y$ , and  $\epsilon_z$  are the emittances of the three decoupled planes in phase space.

The differential equations of motion are numerically integrated over the length of the accelerator element. The equations of motion used for the integration are

$$\frac{d}{dz} \gamma \beta \frac{dx}{dz} = \frac{\Delta E}{L} \frac{1}{mc^2} I_1 \left( \frac{2\pi x}{\gamma_s \beta_s \lambda} \right) \sin \phi \frac{\gamma_s (1 - \beta \beta_s)}{\beta},$$

$$\frac{d\Delta\phi}{dz} = \frac{2\pi}{\lambda} \left( \frac{\beta_s - \beta}{\beta \beta_s} \right), \quad \text{and}$$

$$\frac{d\Delta W}{dz} = \frac{\Delta E}{L} I_0 \left( \frac{2\pi x}{\gamma_s \beta_s \lambda} \right) [\cos \phi - \cos \phi_s],$$

where  $I_\nu(y)$  are solutions of modified Bessel's equations.

Particles are integrated through a typical LAMPF side-coupled accelerator tank (the first tank in the 805-MHz structure). The tank length is 290 cm and the integration step size is 0.1 cm. The integration of 5000 particles takes approximately two hours of CPU time on a VAX 11/780.

It is assumed that the transformation from the initial phase-space coordinates of a particle to the final phase-space coordinates is given by

$$x_{if} = C_i + \sum_{j=1}^6 R_{ij} x_{j0} + \sum_{j=1}^6 \sum_{k=1}^6 T_{ijk} x_{j0} x_{k0}$$

Table 1.

Non-zero new and old first-order transformation matrix elements for an accelerator section.

Element	New	Old
$R_{11} = R_{33}$	$\left( \frac{\gamma_{so} \beta_{so}}{\gamma_{sf} \beta_{sf}} \right)^{\frac{1}{2}} \left( \cosh(k_T L) + \frac{\delta}{k_T} \sinh(k_T L) \right)$	1
$R_{12} = R_{34}$	$\left( \frac{\gamma_{so} \beta_{so}}{\gamma_{sf} \beta_{sf}} \right)^{\frac{1}{2}} \left( \frac{1}{k_T} \sinh(k_T L) \right)$	$\frac{L}{\Delta E \cos \phi_s} \frac{W_{so}}{\cos \phi_s} \ln \left( 1 + \frac{\Delta E \cos \phi_s}{W_{so}} \right)$
$R_{21} = R_{43}$	$\left( \frac{\gamma_{so} \beta_{so}}{\gamma_{sf} \beta_{sf}} \right)^{\frac{1}{2}} \left( k_T \left( 1 - \frac{\delta^2}{k_T^2} \right) \sinh(k_T L) \right)$	0
$R_{22} = R_{44}$	$\left( \frac{\gamma_{so} \beta_{so}}{\gamma_{sf} \beta_{sf}} \right)^{\frac{1}{2}} \left( \cosh(k_T L) - \frac{\delta}{k_T} \sinh(k_T L) \right)$	$\frac{W_{so}}{W_{so} + \Delta E \cos \phi_s}$
$R_{55}$	$\left( \frac{\beta_{sf}}{\beta_{so}} \right) \cos(k_L L)$	1
$R_{56}$	$\left( \frac{\beta_{sf}}{\beta_{so}} \right) \left( \frac{1}{\gamma_{so}^2 k_L} \sin(k_L L) \right)$	0
$R_{66}$	$\left( \frac{\gamma_{so} \beta_{so}^2}{\gamma_{sf} \beta_{sf}^2} \right) (-\gamma_{so}^2 k_L \sin(k_L L))$	$\frac{2\pi}{\lambda} \frac{\Delta E \sin \phi_s}{W_{so} + \Delta E \cos \phi_s}$
$R_{68}$	$\left( \frac{\gamma_{so} \beta_{so}^2}{\gamma_{sf} \beta_{sf}^2} \right) \cos(k_L L)$	$\frac{W_{so}}{W_{so} + \Delta E \cos \phi_s}$

The transformation coefficients  $C_i$ ,  $R_{ij}$ , and  $T_{ijk}$  are found by a least-squares-fit method using the coordinate data generated by the numerical integration of the particle motion. In general, values of  $C_i$  different from zero are an indication of the statistical nature of the random input distribution. The first-order transformation elements,  $R_{ij}$ , can be compared to results obtained by analytical methods. The highest terms from the fit, in this case the second-order terms,  $T_{ijk}$ , give some indication of the error introduced by the truncated, first-order transformation.

The specific case chosen is used because it represents a worst case for LAMPF. The initial energy is the lowest in the side-coupled linac, the emittance the largest, and the tank is relatively long compared to single gaps in the drift-tube linac. Transformation values derived from various methods for this case are compared in Table 2. The three transformation matrices shown in Table 2 are the transformation derived from the fit to numerical integration, the improved transformation now used in LAMPF's version of TRANSPORT, and the transformation used in the earlier version of TRANSPORT. Note the large difference between the elements of the last two matrices for this specific case.

## Conclusion

The first-order accelerator transformation matrix used in the original version of TRANSPORT is acceptable for beams with  $\beta \approx 1$ , but for proton beams with  $\beta < 1$  the transformation in TRANSPORT must be modified. The new accelerator transformation matrix compares well with results from numerical integration. The comparisons are done for typical LAMPF beam parameters with energies greater than 100 MeV.

The new transformation is valid only in the linear region of phase space well away from the separatrix. For applications dealing with beams that fill the rf bucket or needing higher degrees of accuracy, the higher-order transformation terms will need to be developed. Those higher-order approximations could also be compared to results of numerical integration for specific cases to give some estimate of their accuracy.

## References

1. K. L. Brown, F. Rothacker, D. C. Carey, and Ch. Iselin, "TRANSPORT, A Computer Program for Designing Charged Particle Beam Transport Systems," SLAC report SLAC-91, Rev. 2, UC-28.
2. K. Mittag, "On Parameter Optimization for a Linear Accelerator," Kernforschungszentrum Karlsruhe GmbH, report KfK 2555, Jan. 1978.

Table 2.

Numerical values of the first-order transformation matrix for a typical 805-MHz tank in LAMPF's side-coupled linac. The initial energy is 100 MeV, the energy gain is 3.19 MeV over a length of 290 cm, and the synchronous phase is 30 degrees. Only the significant elements are shown. The values are from the results of numerical methods, the new transformation used in LAMPF's new version of TRANSPORT, and the original transformation.

Element	Numerical Integration	Transport (New)	Transport (Old)
$R_{11}$	1.197	1.291	1.000
$R_{12}$	0.306	0.307	0.290
$R_{21}$	1.386	1.427	0.000
$R_{22}$	1.178	1.186	0.997
$R_{33}$	1.197	1.201	1.000
$R_{34}$	0.306	0.307	0.290
$R_{43}$	1.386	1.427	0.000
$R_{44}$	1.178	1.186	0.997
$R_{55}$	0.654	0.641	1.000
$R_{56}$	2.060	2.096	0.000
$R_{65}$	-0.279	-0.281	0.026
$R_{66}$	0.626	0.618	0.997